Safety factor and fatigue life – effective design measures

Many catastrophic failures have resulted from underestimation of design safety and/or fatigue of structures. Failure examples of engineered structures can be seen in many applications such as wind turbines, bridges, engines, and bicycles.

Designs that would have benefited from CAE.

To ensure the structure's ability to carry a load, safety factor and fatigue life are often calculated using detailed analyses, especially when comprehensive testing is costly and time-consuming or impractical. This note will provide a typical example of how the safety factor and fatigue life of a product are effectively calculated using CAE approaches and it will show the design can be improved. We will begin with a review of the two factors to see why they are important in product design.

Keywords: catastrophic failure, safety factor, fatigue life, maximum stress, ultimate strength

1. A quick review of safety factor and fatigue life

Safety factor (SF), which is also known as factor of safety (FoS), is a term describing the load carrying capability of a product beyond the actual load. In other words, SF indicates how much stronger the product is than it usually needs to be for intended loading conditions. Almost all products should deliberately be built stronger than required for normal usage to allow for unexpected loads (emergency or incorrect use) or degradation (material defects and environment). SF is a ratio of yield stress to working stress.

$$SF = \frac{yield\ stress}{working\ stress} \quad (1)$$

where the working stress is calculated from the maximum load that the part should ever bear in service and the yield stress is a property of the material used in the part.

By the above definition, a product with SF = 1.0 will only support the design load and any additional load will cause the product to fail. A product with SF = 3.0 will fail at three times the design load. Depending on applications and materials, SF can vary but must be larger than 1.0. As an example, the SF used in standard automobiles is usually 3.0.

Fatigue life is a term describing how long a product will last before the complete failure. Fatigue is the weakening of a material subjected to repeated loads and is progressive and localised structural damage. There are a number of different factors that can affect the fatigue life of a product including material type; its structure, working temperature and environmental conditions. The stress values that can cause the fatigue damage may be much less than the strength of the material typically quoted as the yield stress or the ultimate stress. The material performance is normally characterised by a S-N graph illustrating the magnitude of a cyclic stress (S) against the logarithmic scales of the number of cycles to failure (N). S-N curves are derived from physical experiments on samples of the material. Shown below are example S-N curves for aluminium and steel.

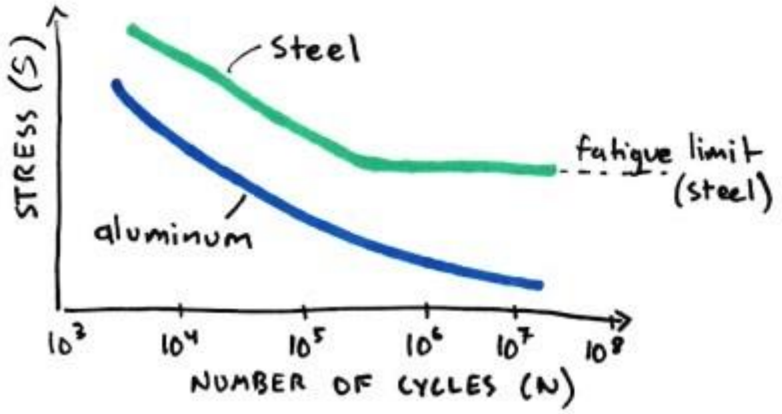

Figure 1: Typical S-N curves of aluminium and steel. Unlike steel, aluminium does not have a fatigue limit. Source: www.firstyearengineer.com

The curve shows how many cycles the material is expected to survive at a given stress. Obviously, as the working stress is increased, the number of cycles that the component can last will decrease. The product is safe to use if, and only if, the working stress is below the curve. With some materials, such as steel, there is what is known as an endurance limit, or a fatigue limit. If the working stress is below the fatigue limit, the component will never fail. If the stress is above the limit, its life will be finite.

The following section provides an example of how SF and fatigue life of a product are analysed by the means of CAE.

2. An example of safety factor and fatigue life analysis

Figure 2: A 3D model of a bike pedal. Courtesy of Rik Elmendorp.

Figure 2 shows the 3-dimensional model of a city bike pedal. A bike pedal usually has many components: pedal body, axle, dust cap, bearings and/or bushings, locking nut, washers, and seals. However, the pedal body and axle are the two critical parts of the assembly that require thorough analyses to evaluate their mechanical behaviours under working conditions.

2.1. Material properties

In this example, the pedal body is made of glass fibre reinforced nylon PA66-GF; AISI 4340 steel is chosen as the material for the axle; AL6061-T6 aluminium is the material of the crank; and, the bearings are made of SUJ2 steel. The respective material properties are shown in Table 1.

Table 1: Material properties of the pedal components used in this analysis.

Properties	Unit	PA66-GF Nylon	AISI 4340 Steel	AL6061 T6	SUJ2 Steel
Density	kg/m^3	1,400	7,850	2,700	7,830
Modulus of Elasticity	GPa	6.1	205	68.9	208
Poisson's ratio		0.34	0.29	0.33	0.3
Yield strength	MPa	120	1,080	276	1,370
Ultimate tensile strength	MPa	140	1,230	310	1,570

Figures 3 and 4 show S-N graphs of AISI 4340 steel and PA66-GF nylon, respectively. These curves will be used for fatigue life calculations in Section 2.6.3. Fatigue life of pedal axle and body. Note that alternating stress = 1/2 (max stress – min stress) and stress ratio R-Ratio = min stress/ max stress.

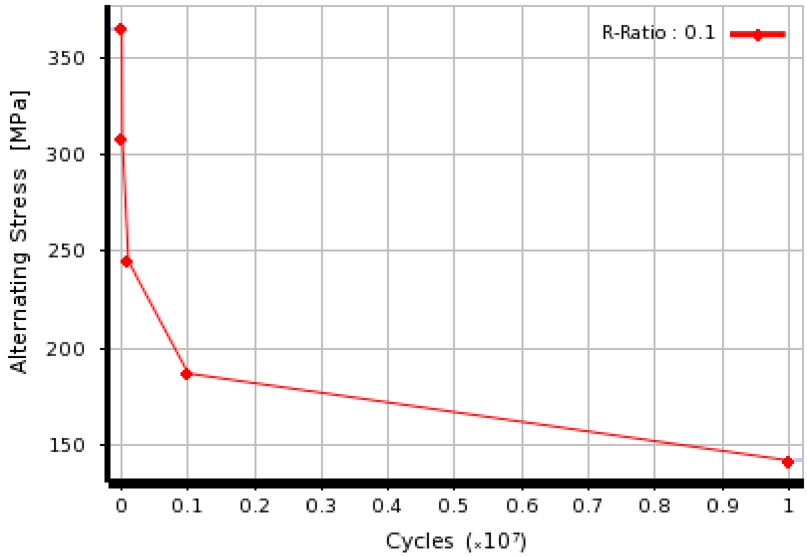

Figure 3: S-N graph of AISI 4340 steel.

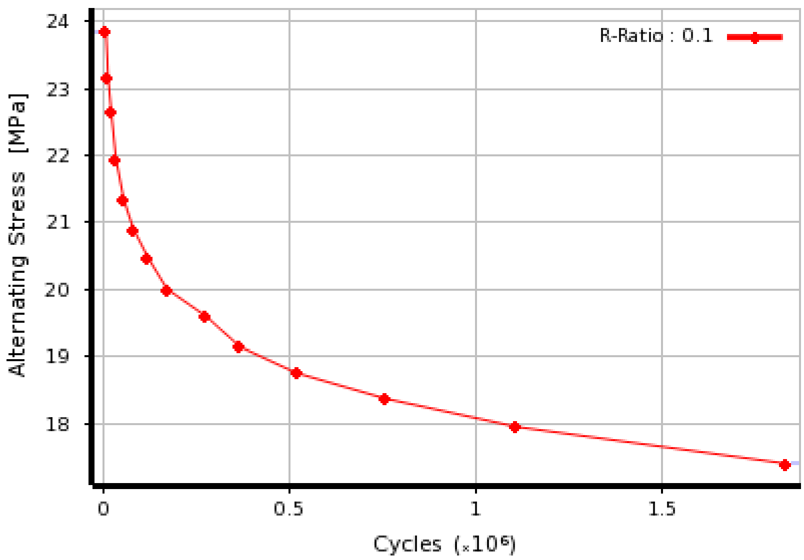

Figure 4: S-N graph of PA66-GF nylon.

2.2. Model simplification

For simplicity, some parts such as pedal rubber pads and reflectors that do not affect the structural strength of the pedal should be removed. Furthermore, small faces and edges are simplified to ensure quality meshes can be achieved. Because the pedal geometry and loading conditions have a plane of symmetry in the vertical plane, we can take advantage of the symmetry boundary conditions in this simulation. As a consequence, computer running time will be reduced and more accurate results can be achieved (because denser meshes can be used with the same computing resource). Figure 5 shows the simplified half model and its components.

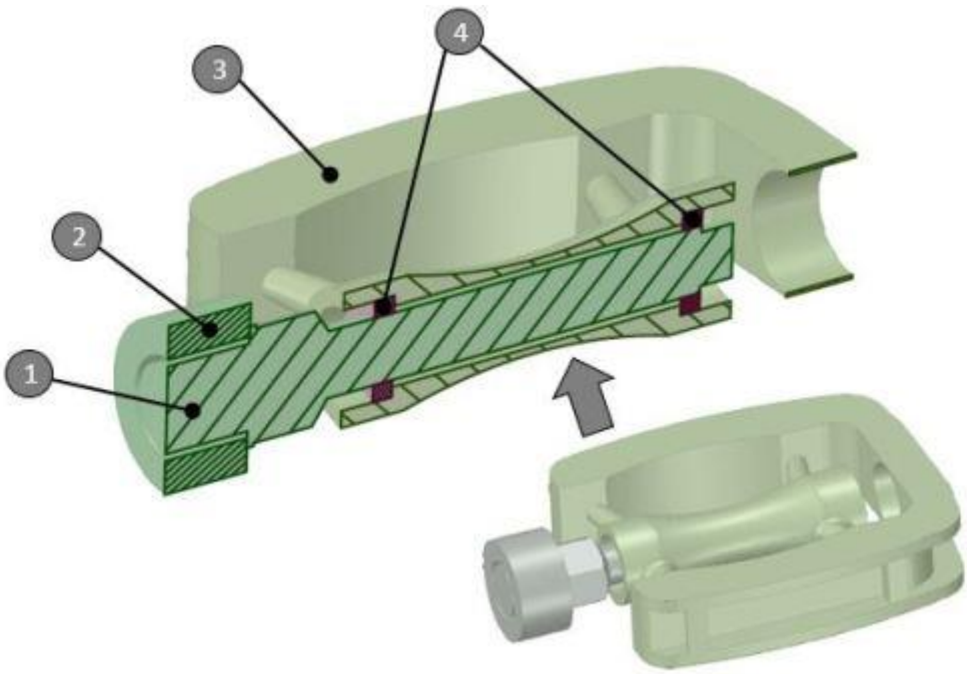

Figure 5: The simplified half model with its components (1. Axle, 2. Crank, 3. Pedal body and 4. Bearings). Note that the cross-section is in the plane of symmetry.

2.3. Boundary conditions

2.3.1. Symmetry boundary

The symmetry boundary condition is applied on the vertical symmetry plane of the model as shown in Figure 6.

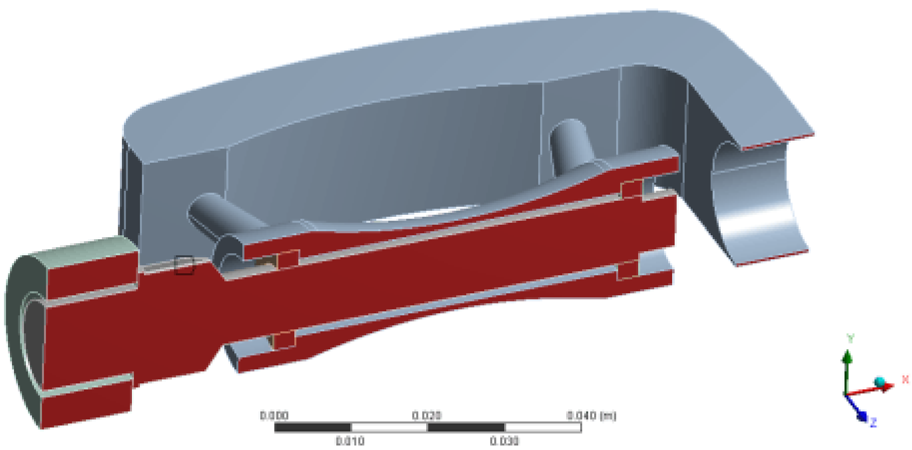

Figure 6: Symmetry boundary condition of the model. The symmetry plane is highlighted in red.

2.3.2. Contacts

There are 3 types of contact conditions applied in this example:

- frictional type for the contact between the axle flange and the crank (with the coefficient of friction being set at 0.61).
- interference fit or rigid bond type between the bearings and the pedal body/housing.
- frictionless type for the contact between the axle and the bearings, as this allows the axle to rotate freely in the housing.

2.3.3. Fixed support

The outer diameter of the simulated crank is fixed in 6-DOFs (degrees of freedom), as shown in Figure 7.

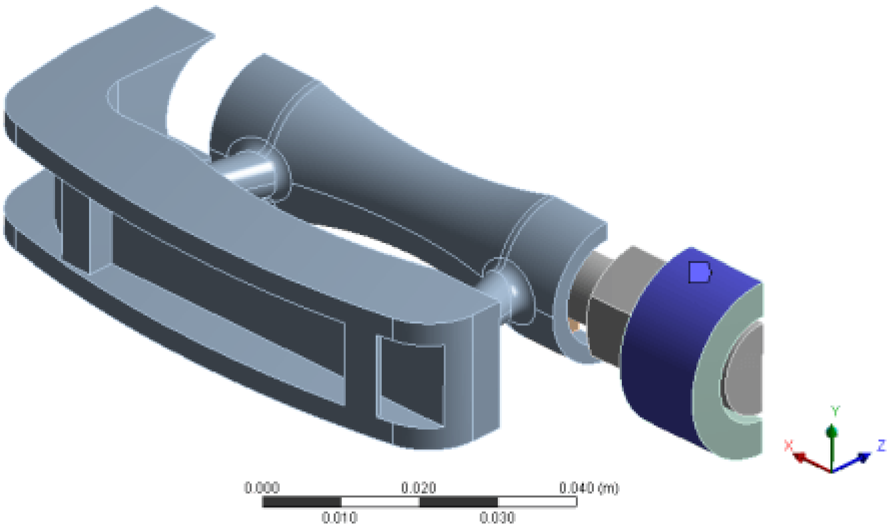

Figure 7: The outer diameter of the simulated crank is fixed in all degrees of freedom. The fixed area is highlighted in purple.

2.3.4. Displacement

The axle thread is set to move freely in the *x*-direction so that its flange can be tightened against the crank. However, the thread displacements are fixed in the *y*– and *z*–directions. This is to satisfy the physics of the thread. Figure 8 shows the displacement conditions applied on the thread face.

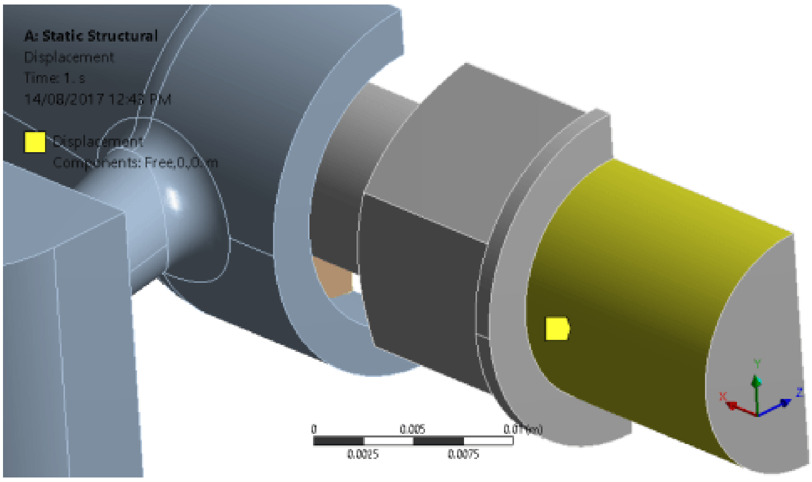

Figure 8: The displacement conditions are applied on the axle thread highlighted in yellow.

2.4. Loading conditions

In essence, the bicycle pedal bears two loads: one is the constant thread preload that occurs when the axle is screwed into the crank. The second load is the bending force applied to the pedal by the rider in order to move the bike forward.

2.4.1. Preload on the axle thread

The pedal axle is normally tightened against the crank at 40 N.m torque (*T*). The resultant preload (*P*) on the axle thread is derived as follows.

$$P = \frac{T}{KD} = \frac{40}{0.2 \times 0.014} = 14{,}286\ N \quad (2)$$

Where *K* = 0.2 is the torque coefficient; *D* = 14 mm is the nominal diameter of the thread; and *T* = 40 N.m. The thread preload is shown in Figure 9.

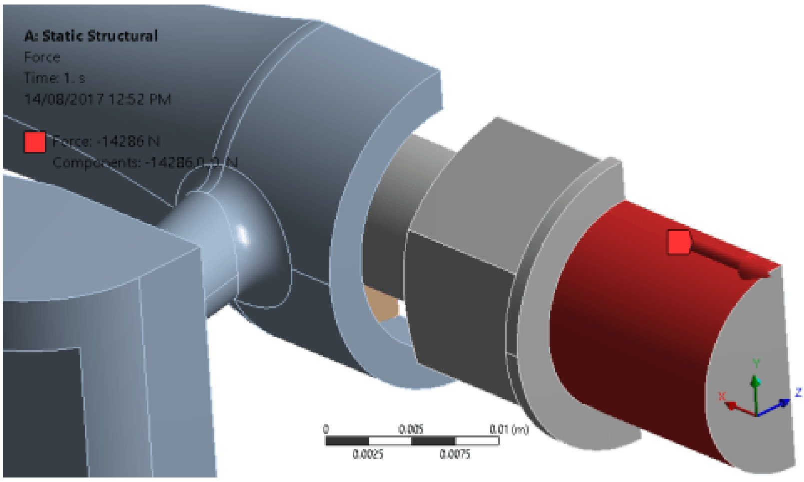

Figure 9: The preload is applied on the thread face highlighted in red.

2.4.2. Pedalling force

When riding, the biker pushes on the pedal to move the bike forward and maximum pedalling force is usually observed with stand-up pedalling. In this example, the rider's weight is assumed to be 80 kg. We further assume that the loading ratio *R* (the ratio of minimum load to maximum load) between two feet is 0.1. The resultant minimum and maximum loads are 7.27 kgf (71.32 N) and 72.73 kgf (713.48 N), respectively. The maximum pedalling force of 713.48 N is chosen for the stress analysis shown in Figure 10.

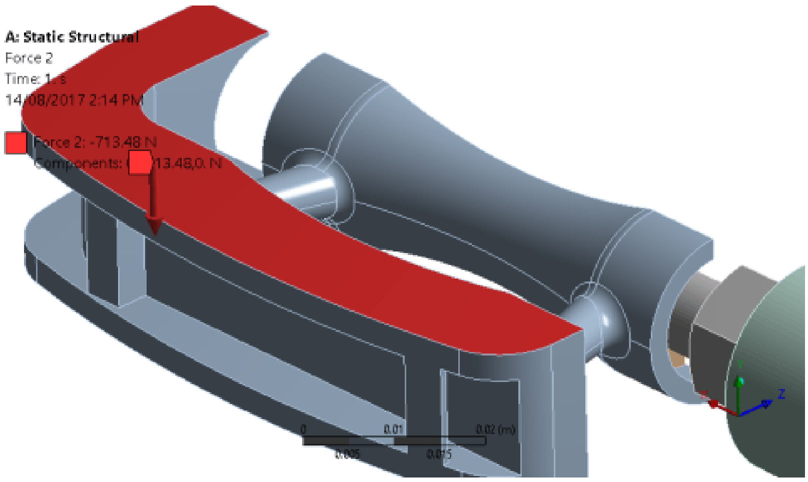

Figure 10: The maximum pedalling force is applied to the pedal body. The active face is highlighted in red.

2.5. Meshing

Figure 11 illustrates the meshing of the model where the element size is set at 1mm. The resulting total number of elements is 82,574.

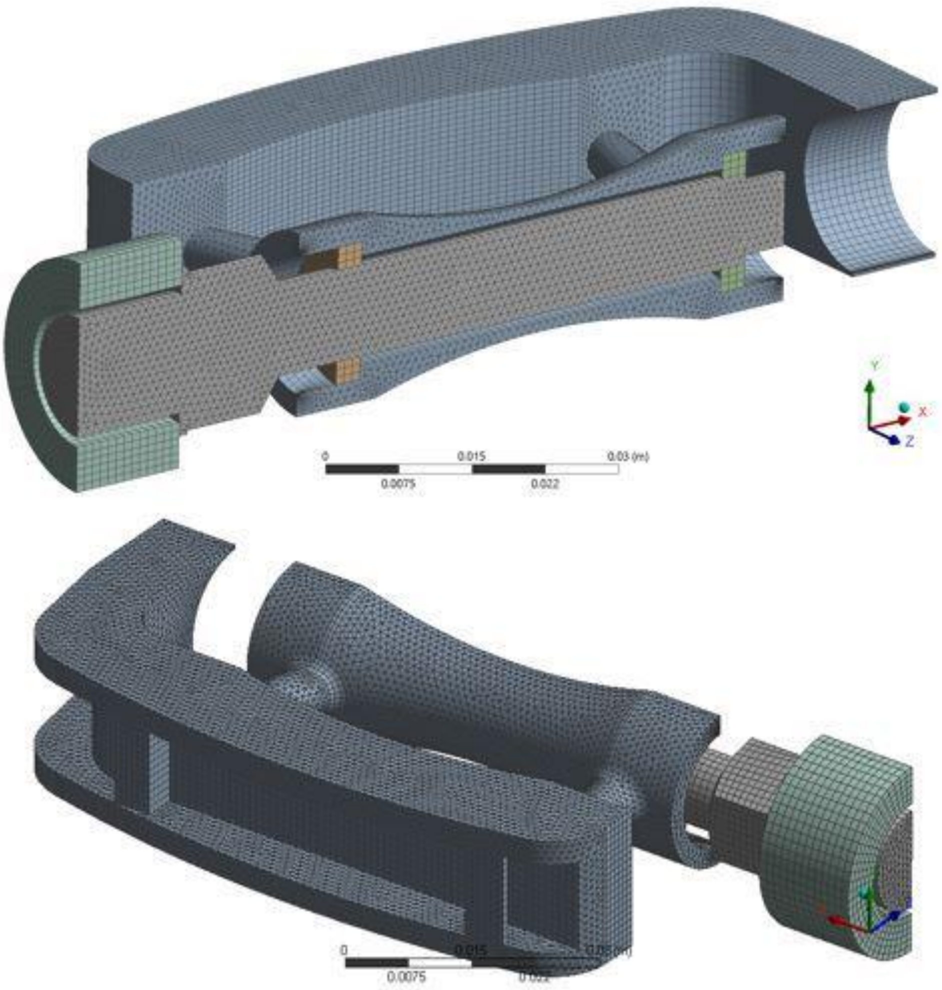

Figure 11: The meshing of the pedal.

2.6. Results and comments

Figure 12 shows the overall stress distribution over the whole model. It can be seen that the maximum stress of the model is located at the stress enhancement step of the axle flange. On the pedal body, the stresses concentrate at the connecting rod from the axle housing to the pedal outer frame.

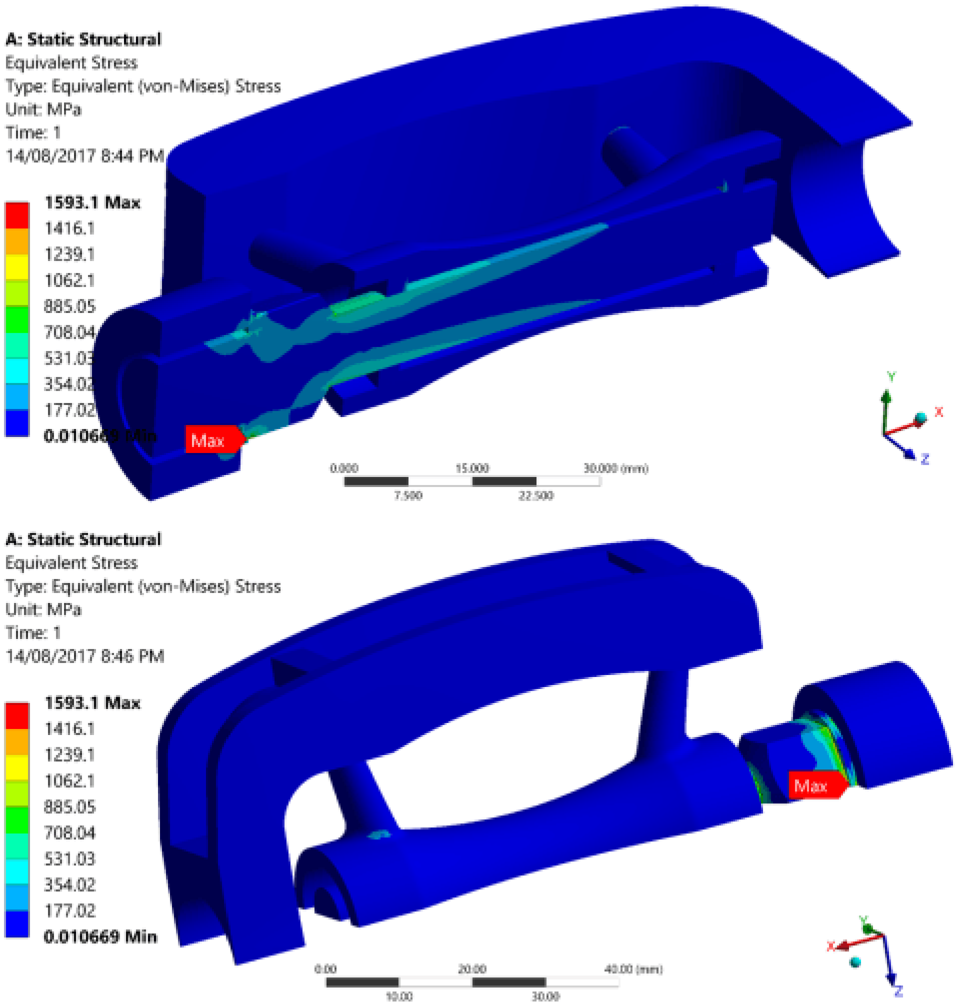

Figure 12: Stress distribution of the whole pedal.

2.6.1. Maximum stress and SF of the axle

The stress distribution of the axle is shown in Figure 13. The maximum stress value is 1,593 MPa which is higher than the chosen steel grade's yield stress of 1,080 MPa. As a result, the flange will be permanently deformed under the maximum design load. *The SF in this case is 0.68 (1,080/1,593). This is less than 1.0, which indicates the axle is not safe for use*. Clearly, the flange thickness must be thickened. The second critical area is the step after the hex where the bending stress is very high at 912.75 MPa. This is close to the yield strength of 1,080 MPa and an increase in the diameter after the hex step would be recommended.

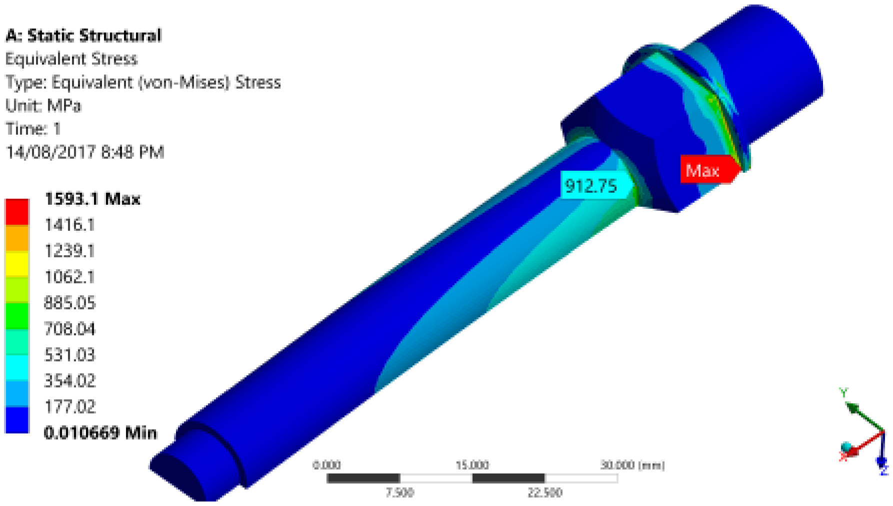

Figure 13: Stress distribution of the axle. Maximum stress value is 1,593.1 MPa. The second critical area has the stress value of 912.75 MPa.

2.6.2. Maximum stress and SF of the pedal body

We can see in Figure 14 below that the high stresses focus on the connecting arms. The maximum stress value of the pedal body is 216.71 MPa which is higher than the nylon material's yield strength of 120 MPa. *The SF of the pedal is 0.55 (120/216.71) which is much less than 1.0.* This means that the arms need to be enlarged so that the maximum stresses on them are less than 120 MPa.

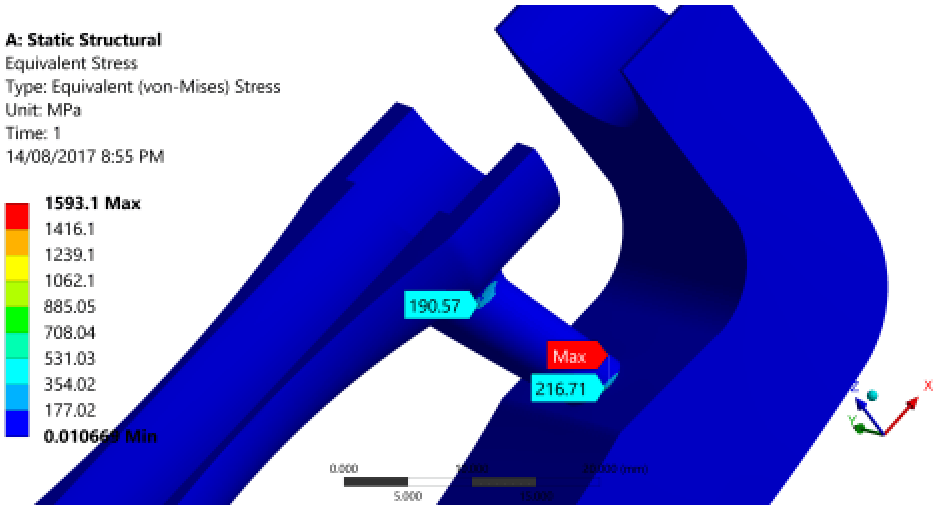

Figure 14: Maximum stress value of the pedal body occurs at the connecting arms.

2.6.3. Fatigue life of pedal axle and body

The fatigue life of the pedal axle and body is predicted in Figure 15. The prediction shows several areas where the fatigue life is zero. This means that the pedal is likely to deform or break when the maximum loading is applied the very first time! The sections that are coloured from **RED** (zero cycles) to **SKY BLUE** (1 million cycles) obviously need to be strengthened.

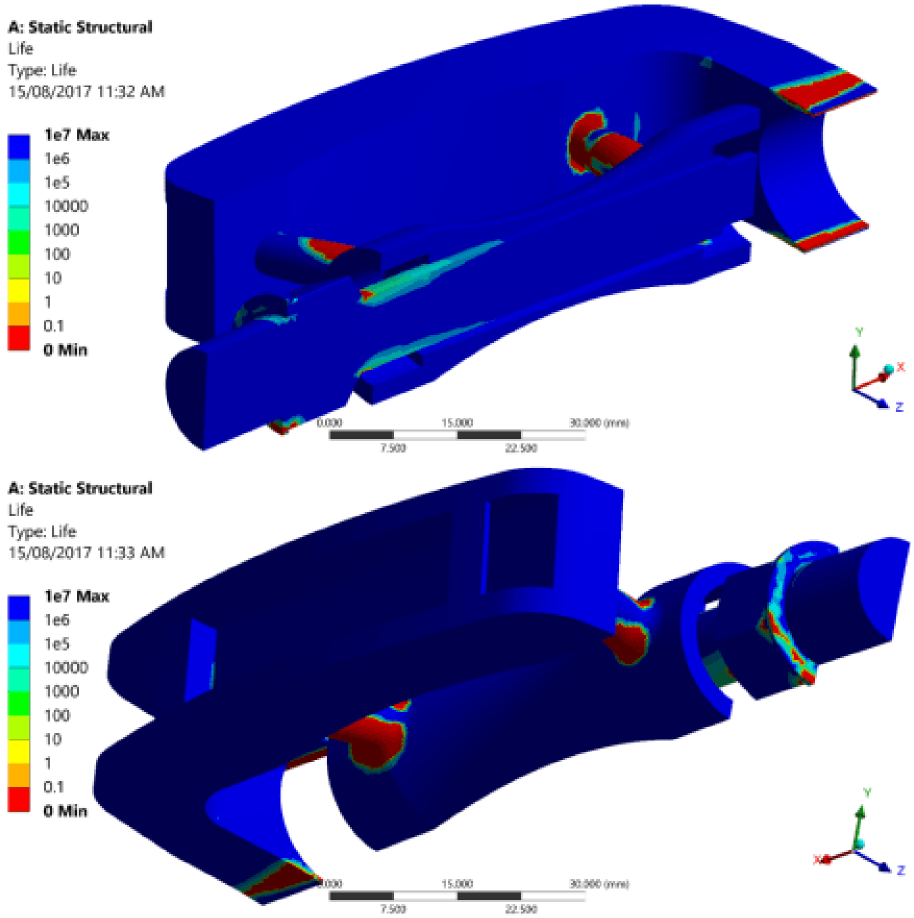

Figure 15: Fatigue life of the pedal body and axle.

2.6.3. Pedal deformation

The maximum deflection of the whole pedal is 2.56 mm as shown in Figure 16. This is significant and would be detectable by the rider.

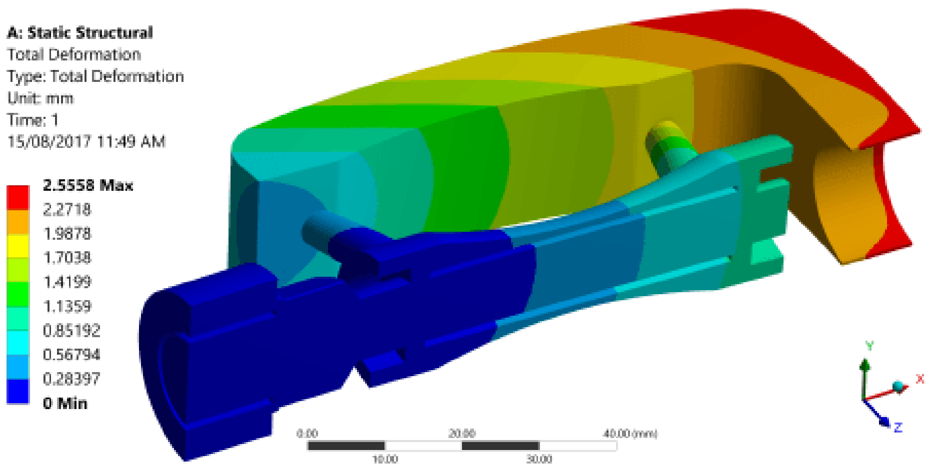

Figure 16: Deformation distribution of the pedal.

2.6.4. Conclusions

As shown in the above analyses, both pedal axle and body are unsafe for use and their fatigue life is too short. The following improvements are suggested:

- For the axle, the flange needs to be thickened and the axle diameter (after the hex) needs to be increased.
- For the pedal body, the connecting arms should be strengthened. This could be achieved by increasing their diameter and by providing a more gradual transition between the arm and the body.
- Incorporate radius fillets wherever a sharp shape transition intensifies the stress.

Through this real life* example, we can see how SF and fatigue life computations can be used to identify weak points and thus improve designs. A similar approach can be applied to more complicated applications and designs.

** Pedal failure could result in serious injury or death, especially with mountain and stand-up bikes. Even a best case injury after a pedal failure is likely to be very sore testicles for a male cyclist. Avoid the pain, use CAE!*

Acknowledgements. The author would like to thank Cliff Walker of Vacmobiles.com for his insightful comments and suggestions that greatly improved the quality of this note. Any errors that remain, of course, are the author's own.

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